Thematics - Combinatorial Number Theory

1. week, September 12: We proved Theorem 1.3 in the lecture notes, the excercies after the theorem is homework. The other homework was the following: If s_t denotes the smallest positive integer in Theorem 1.3. for which the theorem is true in place of $et!$, then $s_t \geq (3^t + 1)/2$.

2. week, September 19: We have finished the chapter "1. Fermat congruence". We have reached the end of the chapter "2. Further Ramsey applications". Excercises 1 and 2 at the end of the chapter are homework, we discussed problem 3 (all 3 solutions are included in the notes). Next week we will continue with the Gallagher sieve.

3. week, September 26: We discussed the homework, especially a lot of time was spent proving Theorem 2.5. We started „3. Gallagher's larger sieve" chapter. So far, we have only look at the statement of the theorems, we have not proved anything from the chapter.

4. week, October 3: We studied the „3. Gallagher's larger sieve" chapter.

5. week, October 10: From the note, "4. We dealt with the chapter "A problem of Diophantus". We watched the video „The Square-Sum Problem". Related to this, you can think about the following until next week: for $n \geq 25$ there is a Hamilton path in the graph whose vertices are $\{1, 2, \ldots, n\}$ and there is an edge between a and a' if $a + a'$ is a square number.

6. week, October 17: We watched the video „Numberphile's Square-Sum Problem was solved!". From the note we studied the chapter "5. Difference sets without squares".

7. week, October 24: We started „6. Sidon sequences" chapter. We gave upper estimates for the maximum size of a Sidon set $\mathcal{A} \subset \{1, 2, \ldots, N\}$. On the other hand, so far we only used the greedy algorithm to give a lower estimate (Theorem 6.4). We will continue from here after the autumn holiday.

8. week, November 7: We finished „6. Sidon sets" chapter, that is, we looked at the lower estimates for the size of the maximal Sidon sets. Pál Erdős proposed numerous conjectures with Sidon sets, related to these, we also looked at the website of T. Bloom, Erdős problems. Unfortunately, today there is no monetary reward for solving the problems... At the end of the lesson, we started „7. Cauchy-Davenport theorem" chapter, where we reached page 68.

9. week, November 14: We finished „7. Cauchy-Davenport theorem" chapter, and we started „8. Combinatorial Nullstelelnsatz" chapter. Here we got to the point where we wrote the polynomial P in the form $P = (x - a_1)Q + R$ and found that the degree of Q is one less than the degree of P, and R has no member containing x_1.

10. week, November 21: We finished „8. Combinatorial Nullstelelnsatz" chapter.

11. week, November 28: We took over „9. Erdős-Ginzburg-Ziv theorem" chapter, and we started the „10. Coloring and density theorems with applications" chapter. Here we have reached the statement of Theorem 10.2, students migh try to figure out the proof before reading. We watched the video „Van der Waerden Theorem", which is about the proof of the theorem (the contents of the videos are not exam material).

12. week, December 5: We gave a lower estimate for the van der Waerden numbers. In the chapter "10. Coloring and density theorems with applications" we proved that there are infinitely many primes by applying the van der Waerden theorem.

13. week, December 12: We have completed Chapter 10. We also went through „11. Behrend's construction" chapter. In the exam, you don't have to be able to say the best lower and upper estimate for $r_k(n)$ (Chapter 10), and I don't ask for the lower estimate for $|\mathcal{A}|$ in the end of Chapter 11 either, but check the calculations at home.